The results of internal auditing work often have some **uncertainty** because resource limitations require internal auditors to use **sampling**. The costs of a complete review of records, transactions, events, performance of control procedures, etc., may exceed both the benefits and the available resources. In these cases, sampling must be done. Thus, internal auditors may apply statistical methods that permit a quantitative assessment of the accuracy and reliability of the sample results. In this way, the internal auditors can evaluate their hypotheses about the matters tested and reduce uncertainty to an acceptable level.

### 7.1 FUNDAMENTALS OF PROBABILITY

**Overview**

1. Probability is important to management decision making because of the unpredictability of future events. According to definitions adopted by some writers,
   
   a. Decision making under conditions of **risk** occurs when the probability distribution of the possible future states of nature is **known**.
   
   b. Decision making under conditions of **uncertainty** occurs when the probability distribution of possible future states of nature is **not known** and must be subjectively determined.

2. Probability provides a method for mathematically expressing doubt or assurance about the occurrence of an event. **The probability of an event varies from 0 to 1**.
   
   a. A probability of 0 means the event cannot occur. A probability of 1 means the event is certain to occur.
   
   b. A probability between 0 and 1 indicates the likelihood of the event’s occurrence. For example, when a fair coin is flipped, the probability that it will land with a given side up is 0.5.

3. **Subjective probabilities** are based on judgment. In business, subjective probability may indicate the degree of confidence a person has that a certain outcome will occur.
   
   a. For example, a manufacturer projecting demand for its product is developing expectations about the overall economy. Management estimates the probability of 1% growth in GDP to be high, 1.5% growth to be medium, and 2% growth to be low.
4. **Objective probabilities** are based on logic or actual experience.
   
a. For example, in rolling fair dice, each face on a single die is equally likely to turn up. Thus, the probability of that event is one in six (.166667).

<table>
<thead>
<tr>
<th>Outcome</th>
<th>Chances</th>
<th>Decimal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Side A</td>
<td>1 in 2</td>
<td>0.5</td>
</tr>
<tr>
<td>Side B</td>
<td>1 in 2</td>
<td>0.5</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>1.0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Outcome</th>
<th>Chances</th>
<th>Decimal</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Dot</td>
<td>1 in 6</td>
<td>0.166667</td>
</tr>
<tr>
<td>2 Dots</td>
<td>1 in 6</td>
<td>0.166667</td>
</tr>
<tr>
<td>3 Dots</td>
<td>1 in 6</td>
<td>0.166667</td>
</tr>
<tr>
<td>4 Dots</td>
<td>1 in 6</td>
<td>0.166667</td>
</tr>
<tr>
<td>5 Dots</td>
<td>1 in 6</td>
<td>0.166667</td>
</tr>
<tr>
<td>6 Dots</td>
<td>1 in 6</td>
<td>0.166667</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>1.000000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Outcome</th>
<th>Chances</th>
<th>Decimal</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 of Clubs</td>
<td>1 in 52</td>
<td>0.019231</td>
</tr>
<tr>
<td>3 of Clubs</td>
<td>1 in 52</td>
<td>0.019231</td>
</tr>
<tr>
<td>4 of Clubs</td>
<td>1 in 52</td>
<td>0.019231</td>
</tr>
<tr>
<td>5 of Clubs</td>
<td>1 in 52</td>
<td>0.019231</td>
</tr>
<tr>
<td>6 of Clubs</td>
<td>1 in 52</td>
<td>0.019231</td>
</tr>
<tr>
<td>7 of Clubs</td>
<td>1 in 52</td>
<td>0.019231</td>
</tr>
<tr>
<td>8 of Clubs</td>
<td>1 in 52</td>
<td>0.019231</td>
</tr>
<tr>
<td>9 of Clubs</td>
<td>1 in 52</td>
<td>0.019231</td>
</tr>
<tr>
<td>Any other card</td>
<td>44 in 52</td>
<td>0.846154</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>1.000000</td>
</tr>
</tbody>
</table>

**Types of Probability**

5. Two events are **mutually exclusive** if they cannot occur simultaneously.
   
a. For example, a flipped coin cannot land with both sides up.

6. Two events are **independent** if the occurrence of one has no effect on the probability of the other.
   
a. For example, when two dice are rolled, the result of one does not affect the result of the other.

7. The **conditional probability** of two events is the probability that one will occur given that the other has already occurred.
   
a. For example, the probability of drawing the queen of clubs from a 52-card deck is 1 in 52 (0.019231). But if the ace of spades has been removed from the deck, the probability of drawing the queen of clubs is higher (1 in 51 or 0.019608).

8. The **joint probability** for two events is the probability that both will occur. It equals the probability (Pr) of the first event times the conditional probability of the second event, given that the first has already occurred.

**EXAMPLE**

If 60% of the students at a university are male, Pr(male) is 6 in 10 (.6). If 1 in 6 of the male students has a B average, Pr(B average given that a student is male) is 1 in 6 (.166667). Thus, the probability that any given student (male or female) selected at random is both male and has a B average is

\[
Pr(male \cap B) = Pr(male) \times Pr(B|male)
\]

\[
= .6 \times .166667
\]

\[
= .10
\]
9. The probability that either one or both of two events will occur equals the sum of their separate probabilities minus their joint probability.

**EXAMPLE**

Assume that one side of a fair coin is heads and the other tails. If two such coins are flipped, the probability that at least one will come up heads is calculated as follows:

\[
\Pr(\text{one or both coins heads}) = \Pr(\text{coin 1 heads}) + \Pr(\text{coin 2 heads}) - \Pr(\text{coin 1 heads and coin 2 heads}) \\
= .5 + .5 - (.5 \times .5) \\
= 1.0 - .25 \\
= .75
\]

**EXAMPLE**

In the earlier example, if 1 in 3 (.33334) of all students, male or female, has a B average [\(\Pr(\text{B average}) = .33334\)], the probability that any given student is male and has a B average is .2 (.6 \times .33334). Accordingly, the probability that any given student either is male or has a B average is

\[
\Pr(\text{male or has B avg.}) = \Pr(\text{male}) + \Pr(\text{B avg.}) - \Pr(\text{B \cap male}) \\
= .6 + .33334 - .2 \\
= .7333
\]

The term \(\Pr(\text{B \cap male})\) must be subtracted to avoid double counting those students who belong to both groups.

10. The sum of the probabilities of all possible mutually exclusive outcomes of a single experiment is 1.0.

**EXAMPLE**

If two fair coins (H = heads, T = tails) are flipped, four outcomes are possible:

<table>
<thead>
<tr>
<th>Coin #1</th>
<th>Coin #2</th>
<th>Probability of This Combination</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>H</td>
<td>.25</td>
</tr>
<tr>
<td>H</td>
<td>T</td>
<td>.25</td>
</tr>
<tr>
<td>T</td>
<td>H</td>
<td>.25</td>
</tr>
<tr>
<td>T</td>
<td>T</td>
<td>.25</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>1.00</strong> (certainty)</td>
</tr>
</tbody>
</table>

**Expected Value**

11. The expected value of a decision (a choice among options) is a weighted average of the payoffs. Each weight is the probability of the related payoff.

a. The highest expected value is optimal.

   1) The decision is under the manager's control.
   2) A state of nature is a future event associated with a payoff.
   3) A payoff is the financial result of (a) the manager's decision and (b) the state of nature.
b. The expected value of a decision is calculated by multiplying the probability of each state of nature by its payoff and adding the products.

**EXAMPLE**

Manager is considering the purchase of two identically priced pieces of property, Bivens Tract and Newnan Tract. Their values will change if a road is built.

The following are the states of nature and their probabilities:

<table>
<thead>
<tr>
<th>Future State of Nature (SN)</th>
<th>Event</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>SN 1</td>
<td>No road is ever built.</td>
<td>.1</td>
</tr>
<tr>
<td>SN 2</td>
<td>A road is built this year.</td>
<td>.2</td>
</tr>
<tr>
<td>SN 3</td>
<td>A road is built more than 1 year from now.</td>
<td>.7</td>
</tr>
</tbody>
</table>

The following are estimates of values for each state of nature:

<table>
<thead>
<tr>
<th>Property</th>
<th>SN 1</th>
<th>SN 2</th>
<th>SN 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bivens Tract</td>
<td>US $10,000</td>
<td>US $40,000</td>
<td>US $35,000</td>
</tr>
<tr>
<td>Newnan Tract</td>
<td>$20,000</td>
<td>$50,000</td>
<td>$30,000</td>
</tr>
</tbody>
</table>

The following are the expected values:

- Bivens Tract: \(0.1(US \: 10,000) + 0.2(US \: 40,000) + 0.7(US \: 35,000) = US \: 33,500\)
- Newnan Tract: \(0.1(US \: 20,000) + 0.2(US \: 50,000) + 0.7(US \: 30,000) = US \: 33,000\)

Thus, the Bivens Tract is the better investment.

This calculation is a payoff table.

**Core Concepts**

- Subjective probabilities are based on judgment. Objective probabilities are based on logic or actual experience.
  - Two events are mutually exclusive if they cannot occur simultaneously.
  - Two events are independent if the occurrence of one has no effect on the probability of the other.
  - The conditional probability of two events is the probability that one will occur given that the other has already occurred.
  - The joint probability for two events is the probability that both will occur.

- The sum of the probabilities of all possible mutually exclusive outcomes of a single experiment is 1.0.
- The expected value of a decision (a choice among options) is a weighted average of the payoffs associated with states of nature (future events). Each weight is the probability of the related payoff.

Stop and review! You have completed the outline for this subunit. Study multiple-choice questions 1 and 2 on page 240.
7.2 STATISTICS

1. A **probability distribution** specifies the values of a random variable and their respective probabilities. Certain standard distributions seem to occur frequently in nature and have proven useful in business. These distributions may be classified according to whether the random variable is discrete or continuous.

   a. If the relative frequency of occurrence of the values of a variable can be specified, the values taken together constitute a function, and the variable is a random variable.

   b. A variable is **discrete** if it can assume only certain values in an interval. For example, the number of customers served is a discrete random variable because fractional customers do not exist. Probability distributions of discrete random variables include the following:

      1) **Uniform distribution.** All outcomes are equally likely, such as the flipping of one coin or even of two coins, as in the example under item 9. in Subunit 7.1.

      2) **Binomial distribution.** Each trial has only two possible outcomes, e.g., accept or reject. This distribution shows the likelihood of each of the possible combinations of trial results. It is useful in quality control.

      3) **Poisson distribution.** An event may occur more than once with random frequency during a given period. Examples of applications are the arrival of customers at a service window and the frequency with which trucks are involved in traffic accidents.

   c. A random variable is **continuous** if no gaps exist in the values it may assume. For example, the weight of an object is a continuous variable because it may be expressed as an unlimited continuum of fractional values as well as whole numbers.

      1) The **normal distribution** is the most important of all distributions and describes many physical phenomena. It has a symmetrical, bell-shaped curve centered about the mean (see Figure 7-1 on the next page).

2. The shape, height, and width of a population’s curve are related to its **measures of central tendency**.

   a. The **mean** is the arithmetic average of a set of numbers.

   b. The **median** is the halfway value if raw data are arranged in numerical order from lowest to highest. Thus, half the values are smaller than the median and half are larger. It is the 50th percentile.

   c. The **mode** is the most frequently occurring value. If all values are unique, no mode exists.

**EXAMPLE**

An investor has eight investments and calculates the measures of central tendency for returns on the portfolio.

**Mean** = Arithmetic average of population values

\[
= \frac{(US \ $43,500 + US \ $52,100 + US \ $19,800 + US \ $41,600 + US \ $52,100 + US \ $66,700 + US \ $33,900 + US \ $54,900)}{8}
\]

= US $364,600 ÷ 8

= US $45,575

**Median** = Midpoint between two central-most population values

Values ranked: US $19,800; US $33,900; US $41,600; US $43,500; US $52,100; US $52,100; US $54,900; US $66,700

\[
= \frac{(US \ $43,500 + US \ $52,100)}{2}
\]

= US $95,600 ÷ 2

= US $47,800

**Mode** = Most frequent value in population

= US $52,100
d. In a **normal distribution**, the mean, median, and mode are the same, and the tails are identical.

![Normal Distribution Diagram](image)

e. **Asymmetrical Distributions**

1) In some frequency distributions, the mean is greater than the mode. The right tail is longer, and the distribution is positively skewed (to the right).

![Positively Skewed Distribution Diagram](image)

a) Accounting distributions tend to be skewed right. For instance, accounts receivable contains many medium- and low-value items and a few high-value items.

2) In some frequency distributions, the median is greater than the mean. The left tail is longer, and the distribution is negatively skewed (to the left).

![Negatively Skewed Distribution Diagram](image)

3) The median is the best estimate of central tendency for many asymmetrical distributions because the median is not biased by extremes.

### Measures of Dispersion

3. A population’s **variability** is the extent to which the individual values of the items are spread about the mean. It is measured by the **standard deviation**.

a. Normal distributions have the following fixed relationships between the area under the curve and the distance from the mean.

<table>
<thead>
<tr>
<th>Distance (±) in Standard Deviations (Confidence Coefficient)</th>
<th>Area under the Curve (Confidence Level)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>68%</td>
</tr>
<tr>
<td>1.64</td>
<td>90%</td>
</tr>
<tr>
<td>1.96</td>
<td>95%</td>
</tr>
<tr>
<td>2.0</td>
<td>95.5%</td>
</tr>
<tr>
<td>2.57</td>
<td>99%</td>
</tr>
<tr>
<td>3.0</td>
<td>99.7%</td>
</tr>
</tbody>
</table>
b. The area under the curve is referred to as the confidence level because a person selecting an item at random from a normally distributed population can be, e.g., 95.5% confident that the item will fall between ±2 standard deviations of the mean (the mean is defined in item 2.a. on page 221).

**EXAMPLE**

A certain species of pine tree has an average adult height of 20 feet, with each standard deviation representing 1 foot. The conclusion from the distribution below is that 68% of all trees of this species will reach a height between 19 and 21 feet (1 standard deviation), 95.5% will be between 18 and 22 feet (2 standard deviations), and 99.7% will be between 17 and 23 feet (3 standard deviations).

![Figure 7-4](image)

**Core Concepts**

- A probability distribution specifies the values of a random variable and their respective probabilities.
- A variable is discrete if it can assume only certain values in an interval. Probability distributions of discrete random variables include the following:
  - Uniform distribution
  - Binomial distribution
  - Poisson distribution
- A random variable is continuous if no gaps exist in the values it may assume.
- The shape, height, and width of a population’s curve are related to its measures of central tendency.
  - The mean is the arithmetic average of a set of numbers. The median is the halfway value if raw data are arranged in numerical order from lowest to the highest. The mode is the most frequently occurring value.
  - In a normal distribution, the mean, median, and mode are the same, and the tails are identical.
- In some frequency distributions, the mean is greater than the mode. The right tail is longer and the distribution is positively skewed (to the right). Accounting distributions tend to be skewed right. In some frequency distributions, the median is greater than the mean. The left tail is longer and the distribution is negatively skewed (to the left).
- A population’s variability is the extent to which the individual values of the items are spread about the mean. It is measured by the standard deviation.

Stop and review! You have completed the outline for this subunit. Study multiple-choice questions 3 and 4 beginning on page 240.
7.3 STATISTICAL SAMPLING

1. Audit testing of every item in a population often is not feasible.
   a. For instance, examining every payment voucher for an entity with annual sales of US $1 billion would require more time than any auditor could devote to it.
      1) Moreover, even if a 100% audit is feasible, the additional reduction of audit risk may not be meaningful.
      2) Fortunately, sampling permits the auditor to state conclusions about the population with measurable reliability.
   b. The main issue in sampling is choosing a sample that is representative of the population. Valid conclusions then may be stated about the population.
   c. The central limit theorem states that, regardless of the distribution of the population from which random samples are drawn, the shape of the sampling distribution of the mean approaches the normal distribution as the sample size is increased.
      1) Thus, whenever a process includes the average of independent samples of the same sample size from the same distribution, the normal distribution can be used as an approximation of that process even if the underlying population is not normally distributed. The central limit theorem explains why the normal distribution is so useful.
      2) The auditor's expectation is that a random sample (i.e., one in which every item has an equal and nonzero chance of being selected) is representative of the population. Thus, the expected value of a random sample of any size is the population mean.
      3) The standard error of the mean is the standard deviation of the distribution of sample means. The standard error is used to compute precision (the confidence interval; see item 4.a. on the next page). The larger the standard error, the wider the interval.
         a) The auditor may estimate the standard error using a pilot sample.

2. Attribute Sampling and Variables Sampling
   a. In attribute sampling, each item in the population has an attribute of interest that is examined by the auditor. This type of sampling is appropriate for tests of controls, i.e., when there are two possible outcomes (compliance/noncompliance).
      1) Because attributes are discrete variables, populations for attribute sampling tend to display the uniform, binomial, and Poisson distributions (see item 1.b. in Subunit 7.2).
   b. In variables sampling, each item in the population has a continuous variable that can be measured. This type of sampling is appropriate for tests of balances, i.e., when the outcome is within a monetary range.
      1) Conclusions about populations for variables sampling are based on the normal distribution [see item 1.c.1) in Subunit 7.2].

3. Parametric vs. Nonparametric Statistics
   a. A parametric statistic (or parameter) is a numerical characteristic of a population computed using all its elements.
      1) The measures of central tendency (mean, median, and mode) are examples of parameters of a population.
b. A nonparametric (distribution-free) statistic is applied to problems for which rank order is known, but the specific distribution is not. For instance, a population may contain various metals ranked in order of hardness but not contain any measure of hardness.

1) Nonparametric statistics are useful when information about the population distribution is lacking.

4. Statistical Calculations Relevant to Sampling

a. Basic precision (also known as the confidence interval) is the range around the sample statistic that is expected to contain the true population parameter.

1) Achieved precision for variables sampling is a function of the standard deviation based on a pilot sample, the specified confidence level, and the sample size.

a) For attribute sampling, the achieved precision is a function of the achieved upper error rate and the sample rate. (Attribute sampling is concerned with error rates, so the measure of interest is the upper rate.)

i) The achieved upper error rate is customarily determined from a standard table. It is the intersection of the sample size and the number of deviations.

ii) The difference between the upper error rate and the sample rate is the achieved precision.

b. The confidence level is the percentage of times that one would expect the sample to adequately represent the population. Thus, a confidence level of 90% should result in samples that adequately represent the population 90% of the time.

1) In other words, given repeated random sampling from a normally distributed population, 90% of the confidence intervals that may be constructed from simple random samples will contain the population mean.

c. Testing a null hypothesis is one method for determining whether the tolerable deviation rate (for attribute testing) or tolerable misstatement (for variables testing) has been exceeded. The null hypothesis is formulated so that rejecting it allows the auditor to make some statement about the population with a specified degree of confidence.

1) For example, a quality control department wants to be 95% confident that a certain component operates properly at all pressure levels below 80 kilograms per square centimeter. The null hypothesis is that the average failure pressure is at least equal to 80 kilograms per square centimeter.

a) If the sample result exceeds a critical value determined for a 95% confidence level, the null hypothesis cannot be rejected.

2) Testing for this condition is far less expensive and time consuming than testing every component.

d. The t-distribution (also known as Student’s distribution) is a special distribution used with small samples (usually fewer than 30) when the population variance is unknown.

1) A t-test can only be applied to a parametric statistic, i.e., some numeric value that is calculated for all items in the population.

2) The t-distribution is useful in business because large samples are often too expensive. For a small sample, the t-statistic (from a t-table) provides a better estimate of the standard deviation than that from a table for the normal distribution.
e. The chi-square distribution is used in testing the fit between actual data and the theoretical distribution. In other words, it tests whether the sample is likely to be from the population, based on a comparison of the sample variance and the estimated population variance. The chi-square test is useful in business for testing hypotheses about populations.

1) If the variance of a process is known and a sample is tested to determine whether it has the same variance, the chi-square statistic may be calculated.

2) A calculated value of the chi-square statistic greater than the critical value in the chi-square table indicates that the sample chosen comes from a population with greater variance than the hypothesized population variance.

a) The chi-square test is appropriately applied to nominal data, meaning data that simply distinguish one from another, as male from female.

**Core Concepts**

- A sample should be representative of the population. Valid conclusions then may be stated about the population.
- The auditor’s expectation is that a random sample (i.e., one in which every item has an equal and nonzero chance of being selected) is representative of the population. Thus, the expected value of a random sample of any size is the population mean. The standard error of the mean is the standard deviation of the distribution of sample means.
  - In attribute sampling, each item in the population has an attribute of interest that is examined by the auditor. This type of sampling is used for tests of controls, i.e., when two outcomes are possible (compliance/noncompliance).
  - In variables sampling, each item in the population has a continuous variable that can be measured. This type of sampling is appropriate for tests of balances, i.e., when the outcome is within a monetary range.
- Basic precision (also known as the confidence interval) is the range around the sample statistic that is expected to contain the true population parameter. Confidence level is the percentage of times that one would expect the sample to adequately represent the population.
- The t-distribution (also known as Student’s distribution) is a special distribution used with small samples (usually fewer than 30) when the population variance is unknown. A t-test can only be applied to a parametric statistic. The t-distribution is useful in business because large samples are often too expensive.
- The chi-square distribution is used in testing the fit between actual data and the theoretical distribution. In other words, it tests whether the sample is likely to be from the population, based on a comparison of the sample variance and the estimated population variance. The chi-square test is useful in business for testing hypotheses concerning populations.

Stop and review! You have completed the outline for this subunit. Study multiple-choice questions 5 through 9 beginning on page 241.

### 7.4 SAMPLING

**Definitions**

1. **Sampling** applies an engagement procedure to fewer than 100% of the items under review for the purpose of drawing a conclusion about some characteristic of the population.

a. **Nonsampling risk** is audit risk not related to sampling. A common audit risk is the auditor’s failure to detect an error in a sample.

1) Nondetection of an error in a sample can be caused by auditor inattention or fatigue. It also can be caused by application of an inappropriate audit procedure, such as looking for the wrong approvals in a sample of documents.
b. **Sampling risk** is the risk that a sample is not representative of the population. An unrepresentative sample may result in an incorrect conclusion.

1) One **advantage of statistical sampling** is that it allows the auditor to quantify sampling risk. An auditor should never attempt to quantify the sampling risk of a nonstatistically drawn sample.

2. **Nonstatistical vs. Statistical Sampling**

a. **Judgment (nonstatistical) sampling** is a subjective approach to determining the sample size and sample selection.

1) This subjectivity is not always a weakness. The internal auditor, based on other work, may be able to test the most material and risky transactions and to emphasize the types of transactions subject to high control risk.

b. **Statistical (probability or random) sampling** is an objective method of determining sample size and selecting the items to be examined.

1) Unlike judgment sampling, it provides a means of quantitatively assessing precision or the allowance for sampling risk (how closely the sample represents the population) and reliability or confidence level (the probability the sample will represent the population).

a) Statistical sampling is applicable to tests of controls (attribute sampling) and substantive testing (variables sampling).

2) For example, testing controls over sales is ideal for random selection. This type of sampling provides evidence about the quality of processing throughout the period.

3) However, a sales cutoff test is an inappropriate use of random selection. The auditor is concerned that the sales journal has been held open to record the next period’s sales. The auditor should select transactions from the latter part of the period and examine supporting evidence to determine whether they were recorded in the proper period.

3. **Types of Sampling Risk**

a. The risk of a **Type I error** is the risk that the auditor will reject a value that is in fact correct. This type of error relates to audit efficiency.

b. The risk of a **Type II error** is the risk that the auditor will accept a value that is in fact incorrect. This type of error relates to audit effectiveness and is therefore the more serious of the two types.

**Levels of Sampling Risk**

4. The internal auditor controls sampling risk by specifying the acceptable levels of its components when developing the sampling plan.

a. For **tests of controls** (an application of attribute sampling), sampling risk includes the following:

1) The risk of assessing control risk too low is the risk that the actual control risk is greater than the assessed level of control risk based on the sample. This risk relates to engagement effectiveness (a Type II error).

2) The risk of assessing control risk too high is the risk that actual control risk is less than the assessed level of control risk based on the sample. This risk relates to engagement efficiency (a Type I error).
b. For **substantive tests** (an application of variables sampling), sampling risk includes the following:

1) **The risk of incorrect acceptance** is the risk that the sample supports the conclusion that the amount tested is not materially misstated when it is materially misstated. This risk relates to engagement effectiveness (a Type II error).

2) **The risk of incorrect rejection** is the risk that the sample supports the conclusion that the amount tested is materially misstated when it is not. This risk relates to engagement efficiency (a Type I error).

   a) If the cost and effort of selecting additional sample items are low, a higher risk of incorrect rejection may be acceptable.

c. The **confidence level** (the reliability level) is the complement of the applicable sampling risk factor.

1) Thus, for a test of controls, if the risk of assessing control risk too low is 5%, the internal auditor’s confidence level is 95% (1.0 – .05).

2) For a substantive test conducted using classical variables sampling, if the risk of incorrect rejection is 5%, the auditor’s confidence level is 95% (1.0 – .05).

**Major Statistical Sampling Methods**

5. **Random sampling** is the most basic statistical sampling method. The term random in this context means that every item in the population has an equal and nonzero chance of being selected. Computer software greatly simplifies this process.

   a. For example, an auditor can enter a range of invoice numbers into a computer-based random number generator. The software ensures that the selection of the invoice numbers is unbiased.

6. **Systematic (interval) sampling** is only appropriate when the auditor has reason to believe that deviations or misstatements are evenly distributed throughout the population.

   a. Systematic sampling divides the population by the sample size and selects every nth item after a random start in the first interval [e.g., population of 1,000 items, sample size of 35, select every 28th item (1,000 ÷ 35 = 28.57)].

   b. Systematic sampling is appropriate when, for instance, an auditor wants to test whether controls were operating throughout an entire year (a random sample might result in all items being selected from a single month).

7. **Stratified sampling** divides the population into subpopulations with similar characteristics. This has the effect of reducing variability within each subpopulation while reducing sample size for a specified confidence level.

   a. A common example is stratifying accounts receivable by outstanding balance. High-monetary-value items can be separated and given extra scrutiny. However, an auditor also considers inherent risk as well as the amounts of monetary items.

8. For **very large populations**, the absolute size of the sample affects the precision of its results more than its size relative to the population. Thus, for populations over 5,000 items, the sample size generally does not increase.

   a. This phenomenon is graphically depicted in Figure 7-5 on the next page for a 95% confidence level, a 1% expected deviation rate, and a 5% tolerable deviation rate:
Core Concepts

- Sampling applies an engagement procedure to fewer than 100% of the items under review for the purpose of drawing a conclusion about some characteristic of the population.
  - Nonsampling risk is audit risk not related to sampling. A common audit risk is the auditor’s failure to detect an error in a sample.
  - Sampling risk is the risk that a sample is not representative of the population.
- Judgment (nonstatistical) sampling is a subjective approach to determining the sample size and sample selection. Statistical (probability or random) sampling is an objective method of determining sample size and selecting the items to be examined.
  - Statistical sampling allows the auditor to quantify sampling risk. An auditor should never attempt to quantify the sampling risk of a nonstatistically drawn sample.
- The risk of a Type I error is the risk that the auditor will reject a value that is in fact correct. This type of error relates to audit efficiency. The risk of a Type II error is the risk that the auditor will accept a value that is in fact incorrect. This type of error relates to audit effectiveness and is therefore the more serious of the two types.
  - For tests of controls (an application of attribute sampling), sampling risk includes the risk of assessing control risk too low (a Type II error) and the risk of assessing control risk too high (a Type I error).
  - For substantive tests (an application of variables sampling), sampling risk includes the risk of incorrect acceptance (a Type II error) and the risk of incorrect rejection (a Type I error).
- The confidence level (reliability level) is the complement of the applicable sampling risk factor. For a test of controls, if the risk of assessing control risk too low is 5%, the internal auditor’s confidence level is 95% (1.0 – .05). For a substantive test conducted using classical variables sampling, if the risk of incorrect rejection is 5%, the auditor’s confidence level is 95% (1.0 – .05).
- Major statistical sampling methods include the following:
  - Sampling is random if every item has an equal and nonzero chance of being selected.
  - Systematic (interval) sampling divides the population by the sample size and selects every nth item after a random start in the first interval.
  - Stratified sampling divides the population into subpopulations with similar characteristics. It reduces variability within each subpopulation and the required sample size.
Stop and review! You have completed the outline for this subunit. Study multiple-choice questions 10 through 15 beginning on page 242.

7.5 ATTRIBUTE SAMPLING

1. Uses
   a. In attribute sampling, each item in the population has an attribute of interest to the auditor, e.g., evidence of proper authorization. Thus, attribute sampling is appropriate for tests of controls, i.e., when two outcomes are possible (compliance or noncompliance).

2. Determining the Sample Size
   a. The sample size for an attribute test is calculated as follows (this formula does NOT have to be memorized for the CIA exam):

   \[
   \text{Sample size for attribute sampling} = \frac{CC^2 \times EDR \times (1 - EDR)}{TDR^2}
   \]

   If:
   - \( CC \) = confidence coefficient
   - \( EDR \) = expected deviation rate
   - \( TDR \) = tolerable deviation rate

   1) The confidence coefficient (a numerator item) is the number of standard deviations associated with the confidence level the auditor has selected.
      a) If the auditor wishes to rely on the tested control, (s)he must increase the confidence level and the sample size.

   2) The estimated deviation rate (a numerator item) is the percentage of items in the population that the auditor believes to be in error.
      a) If the estimated deviation rate is 50% or less, the sample size must increase. Testing a control with such a high estimated deviation rate is inefficient because the auditor cannot rely on it.

   3) The tolerable deviation rate (a denominator item) is the highest allowable percentage of the population that can be in error and still allow the auditor to rely on the tested control.
      a) As the tolerable deviation rate increases, the sample size can be decreased.

   Factors affecting attribute sample size
   - As the confidence coefficient increases, the sample size must increase.
   - As the estimated deviation rate increases, the sample size must increase.
   - As the tolerable deviation rate increases, the sample size can decrease.

3. Calculating the Planned Precision
   a. In attribute sampling, precision (confidence interval) is determined by subtracting the estimated deviation rate from the tolerable deviation rate.

   \[
   \text{Planned precision for an attribute sample:}
   \]

   \[\text{Precision} = \text{Tolerable deviation rate} - \text{Estimated deviation rate}\]

   1) Planned precision and sample size are inversely related. As the required precision decreases (tightens), the sample size must increase.

4. Testing the Sample
   a. Once the sample items have been randomly selected from the population, they are tested for the attribute of interest, and the sample deviation rate (also called the sample rate of occurrence) is calculated.
5. Calculating the Achieved Precision and Interpreting the Results
   a. The achieved degree of precision for an attribute test is calculated as follows (this formula does NOT have to be memorized for the CIA exam):

   **Achieved precision in attribute sampling**

   \[
   \frac{CC^2 \times SDR \times (1 - SDR)}{Sample\ size}
   \]

   If:  
   \[CC = \text{confidence coefficient}\]
   \[SDR = \text{sample deviation rate}\]

   1) The confidence coefficient (a numerator item)
      a) As the auditor increases the confidence level, the precision of the sample widens.

   2) The sample deviation rate (a numerator item)
      a) As the sample deviation rate increases, the precision of the sample widens.

   3) The sample size (a denominator item)
      a) As the sample size increases, the precision of the sample tightens.

   b. The auditor compares planned and achieved precision and decides whether to rely on the tested control.

      1) If the achieved precision is equal to or less than planned precision, the auditor can conclude that the sample is representative of the population. At the level of confidence specified by the auditor, the control can be deemed reliable.

      2) If the achieved precision is greater than planned precision, the auditor must conclude that the tested control cannot be relied upon at the specified confidence level.

6. Two Other Attribute Sampling Methods
   a. **Discovery sampling** is a form of attribute sampling that is appropriate when even a single deviation would be critical.

      1) The occurrence rate is assumed to be at or near 0%, and the method cannot be used to evaluate results statistically if deviations are found in the sample.

      2) The sample size is calculated so that it will include at least one instance of a deviation if deviations occur in the population at a given rate.

   b. The objective of **stop-or-go sampling**, sometimes called sequential sampling, is to reduce the sample size when the auditor believes the error rate in the population is low.

      1) The auditor examines only enough sample items to be able to state that the deviation rate is below a specified rate at a specified level of confidence. If the auditor needs to expand the sample to obtain the desired level of confidence, (s)he can do so in stages.

      2) Because the sample size is not fixed, the internal auditor can achieve the desired result, even if deviations are found, by enlarging the sample sufficiently. In contrast, discovery sampling uses a fixed sample size.

Stop and review! You have completed the outline for this subunit. Study multiple-choice questions 10 through 16 beginning on page 244.
7.6 VARIABLES SAMPLING -- CLASSICAL

1. Uses
   a. Attribute sampling is used for discrete variables. Variables sampling is used for continuous variables, such as weights or monetary amounts. Variables sampling provides information about whether a stated amount (e.g., the balance of accounts receivable) is materially misstated.

   1) Thus, variables sampling is useful for substantive tests. The auditor can determine, at a specified confidence level, a range that includes the true value.

   b. In variables sampling, both the upper and lower limits are relevant (a balance, such as accounts receivable, can be either under- or overstated).

   1) In attribute sampling, auditors are concerned only with the upper deviation limit, i.e., with the risk of assessing control risk too low.

   c. Auditors employ four classical variables sampling techniques:

   1) Unstratified mean-per-unit
   2) Stratified mean-per-unit
   3) Difference estimation
   4) Ratio estimation

   NOTE: Each method is covered in this subunit following a discussion of sample selection and interpretation.

2. Determining the Sample Size
   a. The sample size for a classical variables test is calculated as follows (this formula does NOT have to be memorized for the CIA exam):

   \[
   \text{Sample size for classical variables sampling} = \left( \frac{CC \times ESD \times \text{Population size}}{\text{Tolerable misstatement}} \right)^2
   \]

   If:
   - CC = confidence coefficient
   - ESD = estimated sample deviation

   1) The confidence coefficient (a numerator item)

   a) If the auditor needs a more precise estimate of the tested amount, (s)he must increase the confidence level and the sample size.

   b) The confidence coefficient serves the same function as in attribute sampling. But in variables sampling it corresponds to a range around the calculated amount rather than an estimate of the maximum error rate.

   2) The estimated standard deviation of the population (a numerator item)

   a) The standard deviation is a measure of the variability of the amounts in the population. An increase in the estimated standard deviation increases the sample size. The estimate is based on a pilot sample.

   3) The population size (a numerator item)

   a) The larger the population, the larger the sample.

   4) The tolerable misstatement (a denominator item)

   a) The tolerable misstatement is based on the auditor’s judgment about materiality.

   b) This factor is desired precision. The narrower the precision, the larger the sample.
b. The precision of sample results in a variables sampling application is inversely proportional to the square root of the sample size. Thus, increasing the sample by a factor of four decreases (tightens) the precision by a factor of two.

**Factors affecting classical variables sample size**

<table>
<thead>
<tr>
<th>Factor</th>
<th>Effect on Sample Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>Confidence Coefficient</td>
<td>Increases</td>
</tr>
<tr>
<td>Estimated Standard Deviation</td>
<td>Increases</td>
</tr>
<tr>
<td>Population Size</td>
<td>Increases</td>
</tr>
<tr>
<td>Tolerable Misstatement</td>
<td>Decreases</td>
</tr>
</tbody>
</table>

3. **Calculating the Achieved Precision and Interpreting the Results**

a. The achieved degree of precision for a classical variables test is calculated as follows (this formula does NOT have to be memorized for the CIA exam):

**Achieved precision in classical variables sampling**

\[
\text{Achieved precision} = \frac{CC \times \text{Standard deviation of the population}}{\sqrt{\text{Population size}}}
\]

If: \( CC = \) confidence coefficient

1) The confidence coefficient (a numerator item)
   
   a) As the auditor increases the confidence level, the precision of the sample widens.

2) The standard deviation of the sample (a numerator item)
   
   a) As the standard deviation of the sample increases, the precision of the sample widens.

3) The square root of the sample size (a denominator item)
   
   a) As the sample size increases, the precision of the sample tightens.

b. The auditor compares the achieved precision with planned precision and decides whether the true value of the tested variable is within the stated range.

1) If the achieved precision is smaller (tighter) than planned precision, the auditor can conclude, at the stated confidence level, that the true amount is within the range.

2) For example, an auditor has tested a variables sample with precision of ±4% and a confidence level of 90% and concluded that the true balance in the account is US $1,000,000.

   a) The precision of ±4% gives the boundaries of the computed range: 4% of US $1,000,000 equals US $40,000, resulting in a range of US $960,000 to US $1,040,000.

   b) The auditor can conclude that there is only a 10% chance that the true balance lies outside this range.

4. **Unstratified Mean-per-Unit (MPU) Sampling**

a. Unstratified MPU is appropriate when unit carrying amounts are unknown or the total is inaccurate.

1) Unstratified MPU involves calculating the mean and standard deviation of the observed amounts of the sample items. The mean is then multiplied by the number of items in the population to estimate the population amount.

2) Precision is determined using the mean and standard deviation of the sample.
5. **Stratified Mean-per-Unit (MPU) Sampling**
   a. Stratified MPU is a means of increasing audit efficiency by separating the population into logical groups, usually by various ranges of the tested amount.
      1) By creating multiple populations, the variability within each is reduced, allowing for a smaller overall sample size.
   b. The following is an example of an accounts receivable population divided into strata:

<table>
<thead>
<tr>
<th>Balance Ranges</th>
<th>Number of Accounts</th>
</tr>
</thead>
<tbody>
<tr>
<td>US $100,000 to US $499,999</td>
<td>12</td>
</tr>
<tr>
<td>US $ 10,000 to US $ 99,999</td>
<td>455</td>
</tr>
<tr>
<td>US $ 0 to US $ 9,999</td>
<td>6,806</td>
</tr>
</tbody>
</table>

   1) The auditor can now efficiently gain assurance regarding the recorded total of receivables by testing all of the balances in the top stratum and statistical samples of those in the other two strata.

6. **Difference Estimation**
   a. Difference estimation is a means of estimating the misstatement of an amount by calculating the differences between the observed and recorded amounts for items in the sample. The auditor then calculates the mean difference and multiplies the mean by the number of items in the population.
      1) This method is appropriate only when per-item carrying amounts and their total are known. Also, the auditor need not stratify the population when (a) the population contains many nonzero differences, (b) the number of over- and understatements is roughly equal, and (c) the differences are relatively uniform.
      2) Precision is calculated using the mean and standard deviation of the differences.

7. **Ratio Estimation**
   a. Ratio estimation is a method of estimating the misstatement of an amount by multiplying the recorded total of the population by the ratio of the total observed amount of the sample items to their total recorded amount.
      1) The requirements for efficient difference estimation also apply to ratio estimation. But this method can be used only when all carrying amounts are positive.
         a) Ratio estimation is preferable to unstratified MPU when the standard deviation of the distribution of ratios is less than the standard deviation of the sample item amounts.
         b) Ratio estimation is preferable to difference estimation when differences are not relatively uniform.

Stop and review! You have completed the outline for this subunit. Study multiple-choice questions 17 through 24 beginning on page 247.

7.7 **VARIABLES SAMPLING -- MONETARY-UNIT**

1. **Application**
   a. Monetary-unit sampling (MUS) is also called probability-proportional-to-size, or PPS, sampling. It applies attribute sampling methods to reach a conclusion about the probability of overstating an account balance.
      1) Under MUS, a monetary unit is the sampling unit. Thus, the item containing the sampled monetary unit is selected for testing.
b. MUS is appropriate for a population that may include only a few overstated items, such as inventory and receivables.

1) A systematic selection method is used (every nth monetary unit is selected). The larger an amount in the population, the more likely it will be selected.

2) MUS is not used when the primary engagement objective is to search for understatements, e.g., of liabilities. Moreover, if many misstatements (over- and understatements) are expected, classical variables sampling is more efficient.

c. In contrast, the classical approach to variables sampling is not always appropriate.

1) When only a few differences between recorded and observed amounts are found, difference and ratio estimation sampling may not be efficient.

2) MPU sampling also may be difficult in an unstratified sampling situation.

Stop and review! You have completed the outline for this subunit. Study multiple-choice questions 25 through 27 beginning on page 249.

7.8 SELECTING THE SAMPLING METHOD

Many of the audit sampling questions that a candidate will encounter on the CIA exam focus on selection of the correct method for a given circumstance rather than on the mechanics of applying it. This subunit consists entirely of such questions. Please review Subunits 7.4 through 7.9, paying special attention to the nature and purpose of each sampling method before attempting to answer the questions in this subunit.

Stop and review! You have completed the outline for this subunit. Study multiple-choice questions 33 through 39 beginning on page 250.

7.9 PROCESS CONTROL TECHNIQUES

1. Statistical quality control determines whether a shipment or production run of units lies within acceptable limits. It is also used to determine whether production processes are out of control.

   a. Items are either good or bad, i.e., inside or outside of control limits.

   b. Statistical quality control is based on the binomial distribution.

Acceptance Sampling

2. This method determines the probability that the rate of defective items in a batch is less than a specified level.

   EXAMPLE

   Assume a sample is taken from a population of 500. According to standard acceptance sampling tables, if the sample consists of 25 items and none are defective, the probability is 93% that the population deviation rate is less than 10%. If 60 items are examined and no defectives are found, the probability is 99% that the deviation rate is less than 10%. If two defectives in 60 units are observed, the probability is 96% that the deviation rate is less than 10%.

Statistical Control Charts

3. These are graphic aids for monitoring the status of any process subject to acceptable or unacceptable variations during repeated operations. They also have applications of direct interest to auditors and accountants, for example, (a) unit cost of production, (b) direct labor hours used, (c) ratio of actual expenses to budgeted expenses, (d) number of calls by sales personnel, or (e) total accounts receivable.

4. A control chart consists of three lines plotted on a horizontal time scale.

   a. The center line represents the overall mean or average range for the process being controlled. The other two lines are the upper control limit (UCL) and the lower control limit (LCL).
b. The processes are measured periodically, and the values (X) are plotted on the chart.
   1) If the value falls within the control limits, no action is taken.
   2) If the value falls outside the limits, the result is abnormal, the process is considered out of control, and an investigation is made for possible corrective action.

c. Another advantage of the chart is that it makes trends and cycles visible.

**EXAMPLE**

The chart below depicts 2 weeks of production by a manufacturer who produces a single precision part each day. To be salable, the part can vary from the standard by no more than +/- 0.1 millimeter.

![Statistical Control Chart](image)

The part produced on the 20th had to be scrapped, and changes were made to the equipment to return the process to the controlled state for the following week’s production.

5. Other Chart Types
   a. **P charts** are based on an attribute (acceptable/not acceptable) rather than a measure of a variable. Specifically, it shows the percentage of defects in a sample.
   b. **C charts** also are attribute control charts. They show defects per item.
   c. An **R chart** shows the range of dispersion of a variable, such as size or weight. The center line is the overall mean.
   d. An **X-bar chart** shows the sample mean for a variable. The center line is the average range.

**Variations**

6. Variations in a process parameter may have several causes.
   a. Random variations occur by chance. Present in virtually all processes, they are not correctable because they will not repeat themselves in the same manner. Excessively narrow control limits will result in many investigations of what are simply random fluctuations.
   b. Implementation deviations occur because of human or mechanical failure to achieve target results.
   c. Measurement variations result from errors in the measurements of actual results.
   d. Model fluctuations can be caused by errors in the formulation of a decision model.
   e. Prediction variances result from errors in forecasting data used in a decision model.

**Benchmarks**

7. Establishing control limits based on benchmarks is a common method. A more objective method is to use the concept of expected value. The limits are important because they are the decision criteria for determining whether a deviation will be investigated.
8. An analysis using expected value provides a more objective basis for setting control limits. The limits of controls should be set so that the cost of an investigation is less than or equal to the benefits derived.
   a. The expected costs include investigation cost and the cost of corrective action.
   
   \[
   \text{Total expected cost} = (\text{Probability of being out of control} \times \text{Cost of corrective action}) + (\text{Probability of being in control} \times \text{Investigation cost})
   \]
   b. The benefit of an investigation is the avoidance of the costs of continuing to operate an out-of-control process. The expected value of benefits is the probability of being out of control multiplied by the cost of not being corrected.

9. A Pareto diagram is a bar chart that assists managers in what is commonly called 80:20 analysis.
   a. The 80:20 rule states that 80% of all effects are the result of only 20% of all causes. In the context of quality control, managers optimize their time by focusing their effort on the sources of most problems.
   b. The independent variable, plotted on the x axis, is the factor selected by the manager as the area of interest: department, time period, geographical location, etc. The frequency of occurrence of the defect (dependent variable) is plotted on the y axis.
   1) The occurrences of the independent variable are ranked from highest to lowest, allowing the manager to see at a glance which areas are of most concern.
   c. Following is a Pareto diagram used by a chief administrative officer who wants to know which departments are generating the most travel vouchers that have been rejected because of incomplete documentation.

![Figure 7-7](image_url)
10. A histogram displays a continuous frequency distribution of the independent variable.
   a. Below is a histogram showing the CAO the amount of travel reimbursement delayed by a typical returned travel voucher.

   ![Histogram](image)

   Figure 7-8

11. A fishbone diagram (also called a cause-and-effect diagram or an Ishikawa diagram) is a total quality management process improvement technique. It is useful in studying causation (why the actual and desired situations differ).
   a. This format organizes the analysis of causation and helps to identify possible interactions among causes.
      1) The head of the skeleton contains the statement of the problem.
      2) The principal classifications of causes are represented by lines (bones) drawn diagonally from the heavy horizontal line (the spine).
      3) Smaller horizontal lines are added in their order of probability in each classification.
   b. Below is a generic fishbone diagram.

   ![Fishbone Diagram](image)

   Figure 7-9
Core Concepts

- Statistical quality control is a method of determining whether a shipment or production run of units lies within acceptable limits. It is also used to determine whether production processes are out of control.

- Acceptance sampling determines the probability that the rate of defective items in a batch is less than a specified level.

- Statistical control charts are graphic aids for monitoring the status of any process subject to acceptable or unacceptable variations during repeated operations. A control chart consists of three lines plotted on a horizontal time scale.
  - The center line represents the overall mean or average range for the process being controlled. The other two lines are the upper control limit (UCL) and the lower control limit (LCL).
  - The processes are measured periodically, and the values (X) are plotted on the chart. If the value falls outside the limits, the process is considered out of control, and an investigation is made for possible corrective action.

- A cost-benefit analysis using expected value provides a more objective basis for setting control limits. The limits of controls should be set so that the cost of an investigation is less than or equal to the benefits derived.

\[
\frac{(\text{Probability of being out of control} \times \text{Cost of corrective action})}{(\text{Probability of being in control} \times \text{Investigation cost})} = \text{Total expected cost}
\]

Stop and review! You have completed the outline for this subunit. Study multiple-choice question 40 on page 252.
QUESTIONS
7.1 Fundamentals of Probability

1. An organization uses two major material inputs in its production. To prepare its manufacturing operations budget, the organization has to project the cost changes of these material inputs. The cost changes are independent of one another. The purchasing department provides the following probabilities associated with projected cost changes:

<table>
<thead>
<tr>
<th>Cost Change</th>
<th>Material 1</th>
<th>Material 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>3% increase</td>
<td>.3</td>
<td>.5</td>
</tr>
<tr>
<td>5% increase</td>
<td>.5</td>
<td>.4</td>
</tr>
<tr>
<td>10% increase</td>
<td>.2</td>
<td>.1</td>
</tr>
</tbody>
</table>

The probability that there will be a 3% increase in the cost of both Material 1 and Material 2 is

A. 15%
B. 40%
C. 80%
D. 20%

Answer (A) is correct. (CIA, adapted)

REQUIRED: The probability of joint increases.

DISCUSSION: The joint probability of occurrence of two independent events equals the product of their individual probabilities. The probability that the cost of Material 1 will increase by 3% is .3. The probability that the cost of Material 2 will increase by 3% is .5. The probability that both will occur is .15 (.3 × .5).

Answer (B) is incorrect. It represents the average of the probabilities of a 3% increase in the costs of Material 1 and Material 2. Answer (C) is incorrect. It represents the sum of the probabilities of a 3% increase in the costs of Material 1 and Material 2. Answer (D) is incorrect. It represents the difference between the probabilities of an increase in the costs of Material 1 and Material 2.

2. The internal auditor is attempting to evaluate the potential monetary effect of a control breakdown in the sales cycle. Two controls are involved. The probability that control A will fail is 10%, and the probability that control B will fail is 5%. The controls are independent such that the failure of one does not affect the failure of the other. In evaluating the potential exposure to the organization, the internal auditor determines that the cost of control A’s failure is US $10,000, the cost of control B’s failure is US $25,000, and the cost of failure of both control procedures at the same time is an additional US $200,000. If the control procedures are not made more effective, the expected cost to the organization is

A. US $1,000
B. US $32,250
C. US $2,250
D. US $3,250

Answer (D) is correct. (CIA, adapted)

REQUIRED: The expected cost to the organization.

DISCUSSION: The expected value is the sum of the products of the possible outcomes and their respective probabilities. Moreover, the joint probability that independent events will occur simultaneously is the product of their probabilities. Accordingly, the expected cost is US $3,250 \[ US \text{ } 3,250 = US \text{ } 10,000 \times 0.10 + US \text{ } 25,000 \times 0.05 + US \text{ } 200,000 \times (0.10 \times 0.05) \].

Answer (A) is incorrect. US $1,000 is the cost of a simultaneous failure. Answer (B) is incorrect. US $32,250 assumes that the joint probability is .15, not .005. Answer (C) is incorrect. US $2,250 omits the cost of a simultaneous failure.

7.2 Statistics

3. An organization with 14,344 customers determines that the mean and median accounts receivable balances for the year are US $15,412 and US $10,382, respectively. From this information, the internal auditor can conclude that the distribution of the accounts receivable balances is continuous and

A. Negatively skewed.
B. Positively skewed.
C. Symmetrically skewed.
D. Evenly distributed between the mean and median.

Answer (B) is correct. (CIA, adapted)

REQUIRED: The conclusion drawn from information about the mean and median of accounts receivable.

DISCUSSION: The mean is the arithmetic average, and the median corresponds to the 50th percentile. Thus, half the values are greater and half are smaller. The auditor can conclude that the distribution is positively skewed (to the right) because the mean is greater than the median and the distribution is continuous.

Answer (A) is incorrect. The mean is greater than the median and the distribution is continuous, so the distribution is positively skewed (to the right). Answer (C) is incorrect. "Symmetrically skewed" is a contradiction. A symmetrical distribution is not skewed. In a symmetrical distribution, the mean, median, and mode (the most frequently occurring value) are equal. Answer (D) is incorrect. Distributions spread evenly between two values are uniform distributions.
4. Which of the following statements is true concerning the appropriate measure of central tendency for the frequency distribution of loss experience shown below?

![Frequency (Time) vs. Loss ($000)]

A. The mean, median, and mode are equally appropriate because the distribution is symmetrical.
B. The mode is the most appropriate measure because it considers the dollar amount of the extreme losses.
C. The median is the most appropriate measure because it is not affected by the extreme losses.
D. The mean is the best measure of central tendency because it always lies between the median and mode.

**Answer (C) is correct. (CIA, adapted)**

**REQUIRED:** The appropriate central tendency measure.

**DISCUSSION:** Measures of central tendency are the mean, the median, and the mode. The mean is the arithmetic average, the median is the value above and below which half of the events occur, and the mode is the most frequently occurring value. The median is the best estimate of central tendency for many asymmetrical distributions because the median is not biased by extremes. This frequency distribution of loss is skewed by the extremely high losses. The median, which consists of absolute numbers of events, is unaffected by the magnitude of the greatest losses.

Answer (A) is incorrect. This graph depicts an asymmetrical distribution. Answer (B) is incorrect. The mode does not consider the extreme losses. It is simply the most frequently occurring value. Answer (D) is incorrect. In this population, the median lies between the mean and the mode. This distribution is skewed right because of a few very high loss values. Consequently, the mean is to the right of both the mode and the median.

7.3 Statistical Sampling

5. The variability of a population, as measured by the standard deviation, is the

A. Extent to which the individual values of the items in the population are spread about the mean.
B. Degree of asymmetry of a distribution.
C. Tendency of the means of large samples (at least 30 items) to be normally distributed.
D. Measure of the closeness of a sample estimate to a corresponding population characteristic.

**Answer (A) is correct. (CIA, adapted)**

**REQUIRED:** The definition of standard deviation.

**DISCUSSION:** The standard deviation measures the degree of dispersion of items in a population about its mean.

Answer (B) is incorrect. The dispersion of items in a population is not a function of the degree of asymmetry of the distribution. For example, a distribution may be skewed (positively or negatively) with a large or small standard deviation. Answer (C) is incorrect. The central limit theorem states that the distribution of sample means for large samples should be normally distributed even if the underlying population is not. Answer (D) is incorrect. Precision is the interval about the sample statistic within which the true value is expected to fall.

6. A 90% confidence interval for the mean of a population based on the information in a sample always implies that there is a 90% chance that the

A. Estimate is equal to the true population mean.
B. True population mean is no larger than the largest endpoint of the interval.
C. Standard deviation will not be any greater than 10% of the population mean.
D. True population mean lies within the specified confidence interval.

**Answer (D) is correct. (CIA, adapted)**

**REQUIRED:** The meaning of a confidence interval.

**DISCUSSION:** The confidence level, e.g., 90%, is specified by the auditor. A confidence interval based on the specified confidence level, also called precision, is the range around a sample value that is expected to contain the true population value. In this situation, if the population is normally distributed and repeated simple random samples are taken, the probability is that 90% of the confidence intervals constructed around the sample results will contain the population value.

Answer (A) is incorrect. Computation of a confidence interval permits a statement of the probability that the interval contains the population value. Answer (B) is incorrect. Two-sided confidence intervals are more common. The area in each tail of a two-sided, 90% interval is 5%. Answer (C) is incorrect. The confidence interval is based on the standard deviation, but it has no bearing on the size of the standard deviation.
7. Management of a large computer manufacturer has been much concerned about the consistency across departments in adhering to new and unpopular purchasing guidelines. An internal auditor has a list that rank-orders all departments according to the percentage of purchases that are consistent with the guidelines and indicates which division the department is from. The internal auditor performs a t-test for differences in means on the average rank of departments in divisions A and B to determine whether there is any difference in compliance with the policy and finds that division A (which has more departments) has a significantly higher (i.e., better) average rank than division B. Which one of the following conclusions should be drawn from this analysis?

A. Division A is complying better with the new policy.
B. A random sample of departments should be drawn and the analysis recalculated.
C. A t-test is not valid when the tested groups differ in size.
D. A t-test is inappropriate for this data, and another type of analysis should be used.

Answer (D) is correct. (CIA, adapted)

REQUIRED: The true statement about application of a t-test to rank-ordered data.

DISCUSSION: A t-test can only be applied to a parametric statistic (some numeric value that is calculated for all items in the population). However, rank order is nonparametric. The actual compliance percentages on which the rankings were established would be valid statistics for use with a t-test (they are parametric).

Answer (A) is incorrect. A t-test is not valid in this case. Answer (B) is incorrect. The auditor already has a list of the entire population and no sampling is needed. Answer (C) is incorrect. A t-test can be used with groups that differ in size.

8. An internal auditor is interested in determining whether there is a statistically significant difference among four offices in the proportion of female versus male managers. A Chi-square test is being considered. A principal advantage of this test compared with a t-test in this circumstance is that

A. Generally available software exists for the chi-square test.
B. The chi-square can both detect a relationship and measure its strength.
C. The chi-square can be applied to nominal data.
D. The chi-square is a parametric, and therefore stronger, test.

Answer (C) is correct. (CIA, adapted)

REQUIRED: The principal advantage of the chi-square test over the t-test.

DISCUSSION: The chi-square test is used in determining the goodness of fit between actual data and the theoretical distribution. In other words, it tests whether the sample is likely to be from the population, based on a comparison of the sample variance and the estimated population variance. The chi-square test is appropriately applied to nominal data, meaning data that simply distinguish one item from another, as male from female. The chi-square statistic is calculated and compared with the critical value in the chi-square table.

Answer (A) is incorrect. Software for the t-test is widely available. Answer (B) is incorrect. The chi-square test cannot measure the strength of a relationship. Answer (D) is incorrect. The chi-square test is nonparametric. It is applied to problems in which a parameter is not calculated.

7.4 Sampling

9. The degree to which the auditor is justified in believing that the estimate based on a random sample will fall within a specified range is called

A. Sampling risk.
B. Non-sampling risk.
C. Confidence level.
D. Precision.

Answer (C) is correct. (CIA, adapted)

REQUIRED: The degree to which an auditor is justified in believing an estimate will fall within a specified range.

DISCUSSION: The confidence level is the percentage of times that one would expect the sample to adequately represent the population. Thus, a confidence level of 90% should result in samples that adequately represent the population 90% of the time. In other words, given repeated random sampling from a normally distributed population, 90% of the confidence intervals that may be constructed from simple random samples will contain the population mean.

Answer (A) is incorrect. Sampling risk is the complement of the confidence level. Answer (B) is incorrect. Non-sampling risk is the risk of improperly auditing the sampled items. It cannot be quantified. Answer (D) is incorrect. Precision is the confidence interval.
10. An important difference between a statistical and a judgmental sample is that with a statistical sample,

A. No judgment is required because everything is computed according to a formula.
B. A smaller sample can be used.
C. More accurate results are obtained.
D. Population estimates with measurable reliability can be made.

Answer (D) is correct.  
(CIA, adapted)

REQUIRED: The important difference between a statistical and a judgmental sample.

DISCUSSION: Judgment (nonstatistical) sampling is a subjective approach to determining the sample size and sample selection. This subjectivity is not always a weakness. The internal auditor, based on other work, may be able to test the most material and risky transactions and to emphasize the types of transactions subject to high control risk. Statistical (probability or random) sampling is an objective method of determining sample size and selecting the items to be examined. Unlike judgment sampling, it provides a means of quantitatively assessing precision or the allowance for sampling risk (how closely the sample represents the population) and reliability or confidence level (the probability that the sample will represent the population).

Answer (A) is incorrect. Judgment is needed to determine confidence levels and sample unit definition. Answer (B) is incorrect. A statistical sample may result in either a smaller or larger sample. Answer (C) is incorrect. Either method may produce greater accuracy.

11. In internal auditing sampling applications, Type I and Type II errors may occur. These risks

A. Result directly from the chance that the sample obtained by the internal auditor is unrepresentative of the population.
B. Can be decreased by using more reliable, albeit more expensive, audit procedures.
C. Have a magnitude based only on the economic consequences of incorrect sample-based conclusions.
D. Refer respectively to the risks that (1) internal controls will fail and (2) the resultant error will go undetected.

Answer (A) is correct.  
(CIA, adapted)

REQUIRED: The true statement about Type I and Type II errors.

DISCUSSION: Sampling risk is the risk that a sample is not representative of the population. Type I and Type II errors are types of errors inherent in the practice of sampling. They refer, respectively, to the rejection of a result that is in fact correct (an audit efficiency error) and acceptance of a result that is in fact incorrect (an audit effectiveness error).

Answer (B) is incorrect. Nonsampling risk is dependent on the quality of engagement procedures. Answer (C) is incorrect. These risks do not inherently depend on economic consequences. Answer (D) is incorrect. Audit risk includes control risk and detection risk.

12. An internal auditor wants to select a statistically representative sample from a population of 475 inventory control sheets. Each sheet lists the description, physical count, bar code, and unit cost for 50 inventory items. The auditor uses a random number table to construct the sample; the first two columns are listed below. 14326 is the randomly chosen starting point; the sample’s first item is found on page 143, line 26. (The route used by the internal auditor is down Column A to the top of Column B.)

<table>
<thead>
<tr>
<th>Column A</th>
<th>Column B</th>
</tr>
</thead>
<tbody>
<tr>
<td>75233</td>
<td>06852</td>
</tr>
<tr>
<td>14326</td>
<td>42904</td>
</tr>
<tr>
<td>76562</td>
<td>64854</td>
</tr>
<tr>
<td>28123</td>
<td>04978</td>
</tr>
<tr>
<td>64227</td>
<td>33150</td>
</tr>
<tr>
<td>80938</td>
<td>04301</td>
</tr>
<tr>
<td>22539</td>
<td>41240</td>
</tr>
<tr>
<td>29452</td>
<td>69521</td>
</tr>
</tbody>
</table>

Where is the fifth item in the sample located?

A. Page 809, line 38.
B. Page 429, line 04.
C. Page 331, line 50.
D. Page 068, line 52.

Answer (C) is correct.  
(CIA, adapted)

REQUIRED: The fifth usable number in a random number table.

DISCUSSION: The fifth usable number is on page 331, line 50. The numbers 76562, 64227, 80938, 29452, 06852, 64854, 04978, and 69521 are not usable because either the first three digits exceed 475 or the last two digits exceed 50. The first number in Column A (75233) is not usable not only for this reason but also because it precedes the random start.

Answer (A) is incorrect. Page 809, line 38 is the fifth random number but not the fifth usable number. Answer (B) is incorrect. Page 429, line 04 is the fourth usable number. Answer (D) is incorrect. Page 068, line 52 is not usable since each page has only 50 lines.
13. Using random numbers to select a sample
   A. Is required for a variables sampling plan.
   B. Is likely to result in an unbiased sample.
   C. Results in a representative sample.
   D. Allows auditors to use smaller samples.

   Answer (B) is correct. (CIA, adapted)
   REQUIRED: The reason to use random numbers to select a sample.
   DISCUSSION: Auditors often use random numbers to select sample items because this method enables each sampling unit and each combination of sampling units to have an equal and nonzero probability of selection. Thus, the sample is likely to be unbiased.
   Answer (A) is incorrect. Although random-number sampling may be used for a variables sampling plan, it is not required. Systematic selection is also acceptable unless the population is not randomly organized. Answer (C) is incorrect. The use of random numbers does not always result in a representative sample. Statistical methods allow auditors to estimate the probability that a random sample is not representative. Answer (D) is incorrect. The use of random numbers does not affect sample size.

14. An internal auditor is designing a stratified, mean-per-unit variables sampling plan for items stated in foreign currency units (FCs). To which one of the following strata should the internal auditor allocate the largest proportion of the overall sample size?

<table>
<thead>
<tr>
<th>Number of Items</th>
<th>Expected Mean</th>
<th>Expected Standard Deviation</th>
<th>Total FC Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. 2,000 FC100</td>
<td>FC9</td>
<td>FC200,000</td>
<td></td>
</tr>
<tr>
<td>B. 2,250 FC200</td>
<td>FC4</td>
<td>FC450,000</td>
<td></td>
</tr>
<tr>
<td>C. 3,000 FC80</td>
<td>FC2</td>
<td>FC240,000</td>
<td></td>
</tr>
<tr>
<td>D. 3,100 FC150</td>
<td>FC1</td>
<td>FC465,000</td>
<td></td>
</tr>
</tbody>
</table>

   Answer (A) is correct. (CIA, adapted)
   REQUIRED: The stratum to be allocated the largest proportion of the overall sample size.
   DISCUSSION: Stratified sampling divides the population into subpopulations with similar characteristics. The effect is to reduce variability within each subpopulation compared with the variability in the total population while maintaining the specified confidence level. After stratification, the auditor is likely to devote the most attention to the highest-risk stratum, that is, the one with the largest expected standard deviation.

7.5 Attribute Sampling

15. When planning an attribute sampling application, the difference between the expected error rate and the maximum tolerable error rate is the planned
   A. Precision.
   B. Reliability.
   C. Dispersion.
   D. Skewness.

   Answer (A) is correct. (CIA, adapted)
   REQUIRED: The difference between the expected error rate and the maximum tolerable error rate.
   DISCUSSION: The precision of an attribute sample (also called the confidence interval or allowance for sampling risk) is an interval around the sample statistic that the auditor expects to contain the true value of the population. In attribute sampling (tests of controls), precision is determined by subtracting the expected error rate from the tolerable error rate in the population.
   Answer (B) is incorrect. Reliability is the confidence level. It is the percentage of times that repeated samples will be representative of the population from which they are taken. Answer (C) is incorrect. Dispersion is the degree of variation in a set of values. Answer (D) is incorrect. Skewness is the lack of symmetry in a frequency distribution.
16. The size of a given sample is jointly a result of characteristics of the population of interest and decisions made by the internal auditor. Everything else being equal, sample size will

A. Increase if the internal auditor decides to accept more risk of incorrectly concluding that controls are effective when they are in fact ineffective.

B. Double if the internal auditor finds that the variance of the population is twice as large as was indicated in the pilot sample.

C. Decrease if the internal auditor increases the tolerable rate of deviation.

D. Increase as sampling risk increases.

Answer (C) is correct. (CIA, adapted)

REQUIRED: The true statement about the effect on the sample size resulting from a change in a relevant variable.

DISCUSSION: In an attribute test, the tolerable deviation rate is inversely related to sample size. If it is increased, sample size will decrease.

Answer (A) is incorrect. An increase in allowable risk decreases sample size. Answer (B) is incorrect. Doubling the variability of the population will cause the sample size to more than double. Answer (D) is incorrect. Sampling risk increases as the sample size decreases.

17. An internal auditor is planning to use attribute sampling to test the effectiveness of a specific internal control related to approvals for cash disbursements. In attribute sampling, decreasing the estimated occurrence rate from 5% to 4% while keeping all other sample size planning factors exactly the same would result in a revised sample size that would be

A. Larger.

B. Smaller.

C. Unchanged.

D. Indeterminate.

Answer (B) is correct. (CIA, adapted)

REQUIRED: The sample size effect of decreasing the estimated occurrence rate.

DISCUSSION: In an attribute test, the expected deviation rate is directly related to sample size. If it is decreased, sample size will decrease.

Answer (A) is incorrect. Increasing the expected error rate increases the sample size. Answer (C) is incorrect. Changing one variable while holding all other factors constant changes the sample size. Answer (D) is incorrect. Decreasing the expected error rate while holding all other factors constant decreases the sample size.

18. If all other sample size planning factors were exactly the same in attribute sampling, changing the confidence level from 95% to 90% and changing the desired precision from 2% to 5% would result in a revised sample size that would be

A. Larger.

B. Smaller.

C. Unchanged.

D. Indeterminate.

Answer (B) is correct. (CIA, adapted)

REQUIRED: The sample size effect of decreasing the confidence level and widening the desired precision interval.

DISCUSSION: In an attribute test, the confidence level is directly related, and the precision is inversely related, to sample size. Thus, if the confidence level is reduced and precision is widened, sample size will be smaller.

Answer (A) is incorrect. Increasing C and narrowing P would result in a larger sample size. Answer (C) is incorrect. Decreasing C and widening P decreases the sample size. Answer (D) is incorrect. The revised sample size is determinable.
19. An auditor has to make a number of decisions when using attribute sampling. The term efficiency is used to describe anything that affects sample size. The term effectiveness is used to describe the likelihood that the statistical sample result will be a more accurate estimate of the true population error rate. Assume an auditor expects a control procedure failure rate of 0.5%. The auditor is making a decision on whether to use a 90% or a 95% confidence level and whether to set the tolerable control failure rate at 3% or 4%. Which of the following statements regarding efficiency and effectiveness of an attribute sample is true?

A. Decreasing the confidence level to 90% and decreasing the tolerable control failure rate to 3% will result in both increased efficiency and effectiveness.
B. Decreasing the tolerable failure rate from 4% to 3% will increase audit efficiency.
C. Increasing the confidence level to 95% and decreasing the tolerable control failure rate to 3% will increase audit effectiveness.
D. Increasing the confidence level to 95% will increase audit efficiency.

Answer (B) is correct. (CIA, adapted)

20. An auditor applying a discovery-sampling plan with a 5% risk of overreliance may conclude that there is

A. A 95% probability that the actual rate of occurrence in the population is less than the critical rate if only one exception is found.
B. A 95% probability that the actual rate of occurrence in the population is less than the critical rate if no exceptions are found.
C. A 95% probability that the actual rate of occurrence in the population is less than the critical rate if the occurrence rate in the sample is less than the critical rate.
D. Greater than a 95% probability that the actual rate of occurrence in the population is less than the critical rate if no exceptions are found.

Answer (D) is correct. (CIA, adapted)

21. What is the chief advantage of stop-or-go sampling?

A. The error rate in the population can be projected to within certain precision limits.
B. Stop-or-go sampling may reduce the size of the sample that needs to be taken from a population, thus reducing sampling costs.
C. Stop-or-go sampling allows sampling analysis to be performed on populations that are not homogeneous.
D. Stop-or-go sampling allows the sampler to increase the confidence limits of the analysis without sacrificing precision.

Answer (D) is correct. (CIA, adapted)
### 7.6 Variables Sampling -- Classical

22. In a variables sampling application, which of the following will result when confidence level is changed from 90% to 95%?

A. Standard error of the mean will not be affected.
B. Nonsampling error will decrease.
C. Sample size will increase.
D. Point estimate of the arithmetic mean will increase.

Answer (C) is correct. *(CIA, adapted)*

**REQUIRED:** The effect of raising the confidence level.

**DISCUSSION:** In any sampling application, attribute or variables, an increase in the confidence level requires a larger sample.

Answer (A) is incorrect. The standard error of the mean is the standard deviation of the distribution of sample means. The larger the sample, the lower the degree of variability in the sample. An increase in confidence level from 90% to 95% requires a larger sample. Thus, the standard error of the mean will be affected. Answer (B) is incorrect. By definition, nonsampling error is unaffected by changes in sampling criteria. Answer (D) is incorrect. The estimate of the mean may increase or decrease if sample size changes.

23. An auditor is considering a sample size of 50 to estimate the average amount per invoice in a large trucking company. How would the precision of the sample results be affected if the sample size was increased to 200?

A. The larger sample would be about two times as precise as the smaller sample.
B. The larger sample would be about four times as precise as the smaller sample.
C. Although precision would not be increased that much, a possible downward bias in the estimate of the average per invoice would be corrected.
D. Since both sample sizes are larger than 30, the increase would not have that much of an effect on precision.

Answer (A) is correct. *(CIA, adapted)*

**REQUIRED:** The effect on precision of increasing the sample size.

**DISCUSSION:** The achieved precision of sample results in a variables sampling application is inversely proportional to the square root of the sample size. Thus, increasing the sample by a factor of four decreases (tightens) the precision by a factor of two.

Answer (B) is incorrect. The precision of sample results does not increase at the same rate as sample size. Answer (C) is incorrect. The expected value of the sample mean is the same regardless of sample size. Answer (D) is incorrect. Precision would tighten by a factor of two.

24. If all other factors in a sampling plan are held constant, changing the measure of tolerable misstatement to a smaller value will cause the sample size to be

A. Smaller.
B. Larger.
C. Unchanged.
D. Indeterminate.

Answer (B) is correct. *(CIA, adapted)*

**REQUIRED:** The effect on sample size of decreasing tolerable misstatement.

**DISCUSSION:** The size of the precision interval in a variables test is based upon the tolerable misstatement that is determined by materiality judgments. As this value decreases, for example, because of a decrease in tolerable misstatement, the size of the required sample increases accordingly, and vice versa. Hence, tolerable misstatement (precision) and sample size are inversely related.

25. In a variables sampling application, if the achieved monetary precision range of the statistical sample at a given confidence level is greater than the desired monetary precision range, this is an indication that the

A. Occurrence rate was smaller than expected.
B. Occurrence rate was greater than expected.
C. Standard deviation was less than expected.
D. Standard deviation was greater than expected.

Answer (D) is correct. *(CIA, adapted)*

**REQUIRED:** The implication if the achieved monetary precision range is greater than the desired monetary precision range.

**DISCUSSION:** Three factors determine the achieved precision of a variables sample: the confidence coefficient, the standard deviation of the sample, and the square root of the sample size. If the achieved monetary precision was greater (i.e., wider) than the desired range, then, holding other factors constant, the standard deviation of the sample was greater than expected.

Answer (A) is incorrect. The occurrence rate is relevant to attribute sampling. Answer (B) is incorrect. The occurrence rate is relevant to attribute sampling. Answer (C) is incorrect. The standard deviation was greater than expected.
26. An auditor is using the mean-per-unit method of variables sampling to estimate the correct total value of a group of inventory items. Based on the sample, the auditor estimates, with precision of ±4% and confidence of 90%, that the correct total is US $800,000. Accordingly,

A. There is a 4% chance that the actual correct total is less than US $720,000 or more than US $880,000.

B. The chance that the actual correct total is less than US $768,000 or more than US $832,000 is 10%.

C. The probability that the inventory is not significantly overstated is between 6% and 14%.

D. The inventory is not likely to be overstated by more than 4.4% (US $35,200) or understated by more than 3.6% (US $28,800).

Answer (B) is correct. (CIA, adapted)

REQUIRED: The proper interpretation of the sample results.

DISCUSSION: A 90% confidence level implies that 10% of the time the true population total will be outside the computed range. Precision of ±4% gives the boundaries of the computed range: 4% times US $800,000 equals US $32,000. Hence, the range is US $768,000 to US $832,000.

Answer (A) is incorrect. The precision, not the confidence level, is ±4%. Answer (C) is incorrect. Precision is a range of values, not the probability (confidence level) that the true value will be included within that range. Answer (D) is incorrect. The precision percentage is not multiplied by the confidence percentage.

27. Difference estimation sampling would be appropriate to use to project the monetary error in a population if

A. Subsidiary ledger book balances for some individual inventory items are unknown.

B. Virtually no differences between the individual carrying amounts and the audited amounts exist.

C. A number of nonproportional differences between carrying amounts and audited amounts exist.

D. Observed differences between carrying amounts and audited amounts are proportional to carrying amounts.

Answer (C) is correct. (CIA, adapted)

REQUIRED: The condition for use of difference estimation sampling.

DISCUSSION: Difference estimation of population error entails determining the differences between the audit and carrying amounts for items in the sample, calculating the mean difference, and multiplying the mean by the number of items in the population. This method is used when the population contains sufficient misstatements to provide a reliable sample and when differences between carrying and audit amounts are not proportional. If differences are proportional, ratio estimation is used. A sufficient number of nonproportional errors must exist to generate a reliable sample estimate.

Answer (A) is incorrect. Individual carrying amounts must be known to use difference estimation. Answer (B) is incorrect. A minimum number of differences must be present to use ratio estimation. Answer (D) is incorrect. Ratio estimation is appropriate for proportional differences.

28. Ratio estimation sampling would be inappropriate to use to project the monetary error in a population if

A. The recorded carrying amounts and audited amounts are approximately proportional.

B. A number of observed differences exist between carrying amounts and audited amounts.

C. Observed differences between carrying amounts and audited amounts are proportional to carrying amounts.

D. Subsidiary ledger book balances for some inventory items are unknown.

Answer (D) is correct. (CIA, adapted)

REQUIRED: The inappropriate use of ratio estimation sampling.

DISCUSSION: Ratio estimation is similar to difference estimation except that it estimates the population error by multiplying the carrying amount of the population by the ratio of the total audit amount of the sample items to their total carrying amount. It has been demonstrated that both ratio and difference estimation are reliable and efficient when small errors predominate and the errors are not skewed. Moreover, audit amounts should be proportional to carrying amounts. Consequently, ratio estimation requires that carrying amounts be known.

Answer (A) is incorrect. Proportional relationships tend to support the use of ratio estimation. Answer (B) is incorrect. A minimum number of differences must be present to use ratio estimation. Answer (C) is incorrect. The existence of proportional differences favors the use of ratio estimation.
7.7 Variables Sampling -- Monetary-Unit

29. When an internal auditor uses monetary-unit statistical sampling to examine the total value of invoices, each invoice

A. Has an equal probability of being selected.
B. Can be represented by no more than one monetary unit.
C. Has an unknown probability of being selected.
D. Has a probability proportional to its monetary value of being selected.

Answer (D) is correct. (CIA, adapted)

REQUIRED: The effect of using monetary-unit sampling to examine invoices.

DISCUSSION: Monetary-unit sampling, also called probability-proportional-to-size (PPS) sampling, is a modified version of attribute sampling that relates deviation rates to monetary amounts. It uses the monetary unit as the sampling unit. MUS sampling is appropriate for testing account balances, such as those for inventory and receivables, in which some items may be far larger than others in the population. In effect, it stratifies the population because the larger account balances have a greater chance of being selected. MUS is most useful if few misstatements are expected. Moreover, it is designed to detect overstatements. It is not effective for estimating understatements because the greater the understatment, the less likely the item will be selected. Special design considerations are required if the auditor anticipates understatements or zero or negative balances.

Answer (A) is incorrect. Each monetary unit, but not each invoice, has an equal probability of being selected unless all invoices are for the same amount. Answer (B) is incorrect. It is possible for two or more monetary units to be selected from the same item; e.g., a 4,500 item will be represented by four monetary units if every 1,000th monetary unit is selected. Answer (C) is incorrect. The probability of selection can be calculated using the monetary value of the item and the monetary value of the population.

30. Monetary-unit sampling (MUS) is most useful when the internal auditor

A. Is testing the accounts payable balance.
B. Cannot cumulatively arrange the population items.
C. Expects to find several material misstatements in the sample.
D. Is concerned with overstatements.

Answer (D) is correct. (CIA, adapted)

REQUIRED: The circumstances in which monetary-unit sampling is most useful.

DISCUSSION: MUS, also called probability-proportional-to-size (PPS) sampling, is a modified version of attribute sampling that relates deviation rates to monetary amounts. It uses the monetary unit as the sampling unit. MUS sampling is appropriate for testing account balances, such as those for inventory and receivables, in which some items may be far larger than others in the population. In effect, it stratifies the population because the larger account balances have a greater chance of being selected. MUS is most useful if few misstatements are expected. Moreover, it is designed to detect overstatements. It is not effective for estimating understatements because the greater the understatement, the less likely the item will be selected. Special design considerations are required if the auditor anticipates understatements or zero or negative balances.

Answer (A) is incorrect. An audit of accounts payable is primarily concerned with understatements. Answer (B) is incorrect. The items in the population must be arranged by cumulative monetary total. The first monetary unit is chosen randomly, the second equals the random start plus the sample interval in monetary units, etc. Answer (C) is incorrect. As the expected amount of misstatement increases, the MUS sample size increases. MUS may also overstate the upper misstatement limit when misstatements are found. The result might be rejection of an acceptable balance.

31. An internal auditor is planning to use monetary-unit sampling for testing the monetary value of a large accounts receivable population. The advantages of using monetary-unit sampling (MUS) include all of the following except that it

A. Is an efficient model for establishing that a low error rate population is not materially misstated.
B. Does not require the normal distribution approximation required by variables sampling.
C. Can be applied to a group of accounts because the sampling units are homogenous.
D. Results in a smaller sample size than classical variables sampling for larger numbers of misstatements.

Answer (D) is correct. (CIA, adapted)

REQUIRED: The item not an advantage of MUS.

DISCUSSION: MUS, also called probability-proportional-to-size (PPS) sampling, is a modified version of attribute sampling that relates deviation rates to monetary amounts. It uses a monetary unit as the sampling unit. MUS is appropriate for testing account balances, such as those for inventory and receivables, in which some items may be far larger than others in the population. In effect, it stratifies the population because the larger account balances have a greater chance of being selected. MUS is most useful if few misstatements are expected. Moreover, it is designed to detect overstatements. It is not effective for estimating understatements because the greater the understatement, the less likely the item will be selected. Furthermore, as the number of expected misstatements increases, MUS requires a larger sample size than classical variables sampling.

Answer (A) is incorrect. MUS is efficient when few misstatements are expected. Answer (B) is incorrect. MUS does not assume normally distributed populations. Answer (C) is incorrect. MUS uses monetary units as sampling units.
### 7.8 Selecting the Sampling Method

#### 32. An internal auditor has taken a large sample from a population that is skewed in the sense that it contains a large number of small balances and a small number of large balances. Given this, the internal auditor can conclude

A. The sampling distribution is not normal; thus, PPS sampling based on the Poisson distribution more accurately defines the nature of the population.

B. The sampling distribution is normal; thus, the Z score value can be used in evaluating the sample results.

C. The sampling distribution is not normal; thus, attribute sampling is the only alternative statistical tool that can appropriately be used.

D. None of the answers are correct.

**Answer (B) is correct. (CIA, adapted)**

**REQUIRED:** The auditor’s conclusion about a skewed population.

**DISCUSSION:** The central limit theorem states that, regardless of the distribution of the population from which random samples are taken, the shape of the sampling distribution of the means approaches the normal distribution as the sample size increases. Hence, Z values (the number of standard deviations needed to provide specified levels of confidence) can be used. Z values represent areas under the curve for the standard normal distribution.

Answer (A) is incorrect. The Poisson distribution is applicable to attribute, not variables, sampling. Answer (C) is incorrect. Attribute sampling is not appropriate for testing monetary balances. Answer (D) is incorrect. The sampling distribution is normal, so the Z value can be used in evaluating the results.

### Questions 33 through 35 are based on the following information. An auditor is testing on a company’s large, normally distributed accounts receivable file. The objectives of the audit are to test end-of-period monetary balances and accounts receivable posting exception (error) rates.

#### 33. The accounts receivable file contains a large number of small monetary balances and a small number of large monetary balances, and the auditor expects to find numerous errors in the account balances. The most appropriate sampling technique to estimate the monetary amount of errors is

A. Difference or ratio estimation.

B. Unstratified mean-per-unit.

C. Probability-proportional-to-size.

D. Attribute.

**Answer (A) is correct. (CIA, adapted)**

**REQUIRED:** The most appropriate sampling technique to estimate the dollar amount of errors.

**DISCUSSION:** Difference estimation calculates the average difference between the audit and book values of sample items and multiplies by the number of items in the population. Ratio estimation multiplies the book value of the population by the ratio of the audit value of the sample to the book value. These methods are useful when small errors predominate and the errors are not skewed. If the number of errors is small, a very large sample is required to provide a representative difference between audit and book values.

Answer (B) is incorrect. Mean-per-unit estimation is used to project a total monetary value by multiplying the mean sample value by the number of items in the population. Unstratified means that the population is not divided into subpopulations. This method is inappropriate when many small balance account errors exist. Answer (C) is incorrect. Probability-proportional-to-size sampling is used for estimating monetary values of errors when the expected error frequency is low. Because the sampling unit is the monetary unit, this method increases the likelihood of selecting large items. Answer (D) is incorrect. Attribute sampling does not involve estimation of monetary amounts.

#### 34. The expected population exception rate is 3% for the accounts receivable posting processes. If the auditor has established a 5% tolerable rate, the auditor would use which sampling plan for testing the actual exception rate?

A. Difference or mean per unit estimation.

B. Discovery.

C. Stratified.

D. Attribute.

**Answer (D) is correct. (CIA, adapted)**

**REQUIRED:** The sampling plan used to test the actual exception rate.

**DISCUSSION:** The accounts receivable posting exception rate would be determined using attribute sampling. Attribute sampling is used for applications involving binary (yes-no or right-wrong) propositions. Whether an item has been posted requires a yes-no answer.

Answer (A) is incorrect. Difference or mean estimation is used when sampling for monetary values. Answer (B) is incorrect. Discovery sampling is only used when exception rates are expected to be very low. Answer (C) is incorrect. Stratified sampling arranges populations for more efficient sampling.
35. To test the accounts receivable file to compute an estimated monetary total, the auditor could use any one of the following sampling techniques except

A. Difference or ratio estimation.
B. Unstratified mean-per-unit estimation.
C. Probability proportional to size.
D. Attribute.

Answer (D) is correct. (CIA, adapted)

REQUIRED: The sampling technique that would not be used to compute an estimated dollar total.

DISCUSSION: Attribute sampling is used for applications involving binary (yes-no or right-wrong) propositions. Attribute sampling does not involve estimation of monetary amounts.

Answer (A) is incorrect. Difference or ratio estimation can be used to estimate population dollar values. Both methods involve determining the difference between the audit and book values of items in the sample. Answer (B) is incorrect. Mean-per-unit estimation averages audit values and multiplies them by the units in the population to estimate the account balance. Answer (C) is incorrect. Probability-proportional-to-size sampling uses the monetary as the sampling unit. It is a means of testing account balances.

36. An auditor is conducting a survey of perceptions and beliefs of employees concerning an organization health care plan. The best approach to selecting a sample is to

A. Focus on people who are likely to respond so that a larger sample can be obtained.
B. Focus on managers and supervisors because they can also reflect the opinions of the people in their departments.
C. Use stratified sampling where the strata are defined by marital and family status, age, and salaried/hourly status.
D. Use monetary unit sampling according to employee salaries.

Answer (C) is correct. (CIA, adapted)

REQUIRED: The best approach for sampling employee beliefs.

DISCUSSION: Stratified sampling divides a population into subpopulations, thereby permitting the application of different techniques to each stratum. This approach reduces the effect of high variability if the strata are selected so that variability among the strata is greater than variability within each stratum. For example, one expects to find greater similarities among married people than between married people and unmarried people.

Answer (A) is incorrect. This convenience sample is likely to emphasize people with the time to respond at the expense of employees who are too busy with company work to respond. Answer (B) is incorrect. Managers and supervisors often do not have the same needs and perceptions as their subordinates and also often misperceive the views of employees. Answer (D) is incorrect. The survey tests perceptions and beliefs, not monetary amounts.

37. When an internal auditor’s sampling objective is to obtain a measurable assurance that a sample will contain at least one occurrence of a specific critical exception existing in a population, the sampling approach to use is

A. Random.
B. Discovery.
C. Probability-proportional-to-size.
D. Variables.

Answer (B) is correct. (CIA, adapted)

REQUIRED: The sampling approach used to obtain a measurable assurance that a sample will contain at least one occurrence of a specific critical exception.

DISCUSSION: Discovery sampling is a form of attribute sampling used to identify critical deviations in a population. The occurrence rate is assumed to be at or near 0%, and the method cannot be used to evaluate results statistically if deviations are found in the sample. Hence, discovery sampling is used for tests of controls, but it is appropriate only when one deviation is critical. The sample size is calculated so that the sample will contain at least one example of a deviation if it occurs in the population at a given rate.

Answer (A) is incorrect. Random sampling is a method used to choose the sample. Answer (C) is incorrect. Probability-proportional-to-size (monetary-unit) sampling is a modified version of attribute sampling that relates deviation rates to monetary amounts. Answer (D) is incorrect. Variables sampling is used to estimate the value of a population, not the occurrence rate of deviations.
38. Variability of the monetary amount of individual items in a population affects sample size in which of the following sampling plans?

A. Attribute sampling.
B. Monetary-unit sampling.
C. Mean-per-unit sampling.
D. Discovery sampling.

Answer (C) is correct. *(CIA, adapted)*

**REQUIRED:** The sampling plan in which variability of the monetary amount of individual items affects the sample size.

**DISCUSSION:** Variables sampling applies to monetary values or other quantities in contrast with the binary propositions tested by attribute sampling. Mean-per-unit sampling is a form of variables sampling. It averages the observed values of the sample items and multiplies the average by the number of items in the population to estimate the population value. Precision is then determined. The formula for the size of a mean-per-unit sample equals the square of the confidence coefficient times the square of the standard deviation, with the product divided by the square of the per-unit precision. The standard deviation measures variability. Thus, the larger the standard deviation (a numerator item), the larger the sample size that is required to achieve specified levels of precision and confidence.

Answer (A) is incorrect. Attribute sampling tests binary (yes/no) propositions. It is not used for tests of monetary amounts, so the variability of monetary amounts is not an issue in determining sample size. Answer (B) is incorrect. Monetary-unit (probability-proportional-to-size) sampling neutralizes variability by defining the sampling unit as an individual monetary unit. Answer (D) is incorrect. The objective of discovery sampling is to select items until at least one item is discovered with a particular characteristic, such as evidence of fraud.

### 7.9 Process Control Techniques

39. A health insurer uses a computer application to monitor physician bill amounts for various surgical procedures. This program allows the organization to better control reimbursement rates. The X-bar chart below is an example of the output from this application.

```
Upper Limit

Expected Mean

Lower Limit
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Select the interpretation that best explains the data plotted on the chart.

A. Random variation.
B. Abnormal variation.
C. Normal variation.
D. Cyclic variation.

Answer (B) is correct. *(CIA, adapted)*

**REQUIRED:** The interpretation that best explains the data plotted on the chart.

**DISCUSSION:** Statistical quality control charts are graphic aids for monitoring the status of any process subject to random variations. For example, an X-bar chart depicts the sample means for a variable. If the values fall within the upper and lower control limits, no action is taken. Accordingly, values outside these limits are abnormal and should be investigated for possible corrective action.

Answer (A) is incorrect. Random variations should fall within realistically determined control limits. Answer (C) is incorrect. Normal variations should fall within realistically determined control limits. Answer (D) is incorrect. In time series analysis, cyclic variation is the fluctuation in the value of a variable caused by change in the level of general business activity.
Questions 40 and 41 are based on the following information. An organization has collected data on the complaints made by personal computer users and has categorized the complaints.

40. Using the information collected, the organization should focus on

A. The total number of personal computer complaints that occurred.
B. The number of computer complaints associated with CD-ROM problems and new software usage.
C. The number of computer complaints associated with the lack of user knowledge and hardware problems.
D. The cost to alleviate all computer complaints.

Answer (C) is correct. (CIA, adapted)

REQUIRED: The organization’s focus based on the data.
DISCUSSION: Complaints based on lack of user knowledge and hardware problems are by far the most frequent according to this chart. Consequently, the company should devote its resources primarily to these issues.
Answer (A) is incorrect. More detailed information is not available. The Pareto diagram does not focus on the total quantity of computer complaints. Answer (B) is incorrect. Complaints about CD-ROMs and software are infrequent. Answer (D) is incorrect. Cost information is not provided.

41. The chart displays the

A. Arithmetic mean of each computer complaint.
B. Relative frequency of each computer complaint.
C. Median of each computer complaint.
D. Absolute frequency of each computer complaint.

Answer (D) is correct. (CIA, adapted)

REQUIRED: The information displayed.
DISCUSSION: This Pareto diagram depicts the frequencies of complaints in absolute terms. It displays the actual number of each type of complaint. The chart does not display arithmetic means, relative frequencies, or medians of each type of complaint.